

# Snow College Mathematics Contest

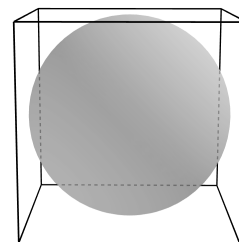
April 3, 2018

Senior Division: Grades 10-12

Form: **T**

Bubble in the single best choice for each question you choose to answer.

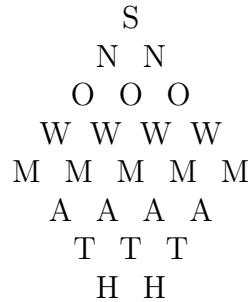
- If  $\log_{10} 5 = 0.699$  what is  $\log_{10} 500$ ?
  - 2.699
  - 5.699
  - 6.99
  - 69.9
  - 500.699
- Three dice are rolled. What is the probability that none of them show a 1 or 2?
  - $\frac{1}{27}$
  - $\frac{8}{27}$
  - $\frac{18}{27}$
  - $\frac{19}{27}$
  - $\frac{26}{27}$
- The number of spots on a particular normal Dalmatian dog is divisible by 3. When the number of spots is divided by the number of legs, a remainder of 3 results. The spots can also be divided by the total of legs, ears, eyes, and tail to leave a remainder of 6. What is the minimum number of spots?
  - 12
  - 15
  - 18
  - 27
  - 36
- Define  $3\mathbb{Z} = \{\dots, -9, -6, -3, 0, 3, 6, 9, \dots\}$ . If we define primes in  $3\mathbb{Z}$  as those positive numbers that cannot be expressed as products of smaller positive elements of the set, what is the sum of the first three positive *composite* (non-prime) numbers in  $3\mathbb{Z}$ ?
  - 9
  - 18
  - 24
  - 36
  - 54
- An equilateral triangle has a side of length  $a$ . What is the area of the largest circle which can be drawn within this triangle?
  - $\frac{\pi}{18}a^2$
  - $\sqrt{3}\pi a^2$
  - $2\sqrt{3}\pi a^2$
  - $\frac{\pi}{12}a^2$
  - $\frac{3}{2}\sqrt{3}a^2$
- A sphere is inscribed in a cube. What is the ratio of the volume of the sphere to the volume of the cube?
  - $\frac{\pi}{6}$
  - $\frac{2\pi}{3}$
  - $\frac{\pi}{8}$
  - $\frac{6}{\pi}$
  - $\frac{\pi}{2}$



7. Find the measure of an angle that is both the complement of  $\angle A$  and the supplement of  $\angle B$  if  $m\angle A + m\angle B = 236^\circ$ .
- (A)  $17^\circ$   
 (B)  $34^\circ$   
 (C)  $45^\circ$   
 (D)  $59^\circ$   
 (E)  $67.5^\circ$
8. Simplify.
- $(\log_{624} 625) (\log_{623} 624) \cdots (\log_6 7) (\log_5 6)$
- (A) 2  
 (B) 2.5  
 (C) 4  
 (D) 5  
 (E) 6
9. A circle and a square have the same perimeter. Then
- (A) their areas are equal.  
 (B) the area of the circle is the greater.  
 (C) the area of the square is the greater.  
 (D) the area of the circle is  $\pi$  times the area of the square.  
 (E) None of these
10. How many two-digit whole numbers are exactly 7 times the sum of their digits?
- (A) 0  
 (B) 1  
 (C) 2  
 (D) 3  
 (E) 4
11. Convert the repeating decimal into a fraction. After reducing to lowest terms, find the difference between the denominator and the numerator.  $0.60\overline{60}$
- (A) 13  
 (B) 33  
 (C) 39  
 (D) 47  
 (E) 60
12. Tau, who loves eating 2 pieces of pi, discovered that when the digits of a three-digit natural number are rearranged to form a second number, the difference between the two numbers is usually divisible by \_\_\_\_.
- (A) 2  
 (B) 4  
 (C) 5  
 (D) 6  
 (E) 9
13. Given three distinct one-digit numbers (i.e., from the set  $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ ), what is the probability that two of them add up to the third one?
- (A)  $\frac{1}{10}$   
 (B)  $\frac{4}{21}$   
 (C)  $\frac{1}{3}$   
 (D)  $\frac{2}{3}$   
 (E)  $\frac{5}{27}$

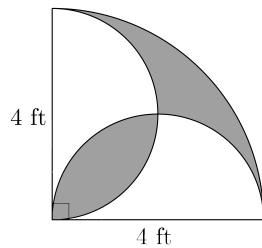
14. Given these rules, how many different paths can you take to spell SNOWMATH?
- Begin at the top
  - Move only down
  - For each move, go to one of the letters directly below

- (A) 2  
 (B) 10  
 (C) 20  
 (D) 35  
 (E) 70



15. Two semicircles are in a quarter-circle. What is the area of the shaded region in  $\text{ft}^2$ ?

- (A)  $2\pi - 4$   
 (B)  $4\pi - 8$   
 (C)  $4\pi - 4$   
 (D)  $2\pi + 4$   
 (E)  $2\pi - 2$



16. Towns A, B, and C are at the corners of a triangle with equal sides. A car travels at constant speed from A to B at 30 mph, from B to C at 40 mph, and from C back to A at 60 mph. What is the average speed for the round trip?

- (A) 40 mph  
 (B) 43 mph  
 (C) 45 mph  
 (D) 48 mph  
 (E) 50 mph

17. Matrix  $X$  is an inverse to matrix  $A$  if  $AX = XA = I$ . Which of the following **cannot** have an inverse matrix?

- (A)  $\begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$   
 (B)  $\begin{bmatrix} 3 & 1 \\ 4 & 1 \end{bmatrix}$   
 (C)  $\begin{bmatrix} 3 & -5 \\ -6 & 10 \end{bmatrix}$   
 (D)  $\begin{bmatrix} 5 & 0 \\ 0 & -2 \end{bmatrix}$   
 (E)  $\begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix}$

18. How many different positive integers less than 2018 are multiples of 20 or 18 (inclusive)?

- (A) 198  
 (B) 201  
 (C) 207  
 (D) 212  
 (E) 252

19. What is  $2018 \pmod{26}$ ?

- (A) 77  
 (B) 16  
 (C) 18  
 (D) 0  
 (E) 1992

20. Driving on a certain street, Prof. B can hit stoplights green 90% of the time. What is the probability that he will hit exactly two of the next three lights green, assuming independent events?

- (A) 0.81  
 (B) 0.081  
 (C) 0.243  
 (D) 0.600  
 (E) 0.900

21. At a certain business, packages are delivered through a square delivery chute, with each side of the chute measuring 3 ft. Suppose a rectangular box of dimensions  $6\text{ ft} \times 4\text{ ft} \times x$  is to be delivered through the chute. What is the maximum value of  $x$ ?

- (A)  $\frac{1}{2}$
- (B)  $2\sqrt{3} - 3$
- (C)  $3\sqrt{2} - 4$
- (D)  $\sqrt{2} - 1$
- (E)  $2\sqrt{5} - 4$

22. If  $f(x) = 3x - 2$ , find  $f(f(f(3)))$ .

- (A) 19
- (B) 55
- (C) 75
- (D) 107
- (E) 163

23. Suppose  $f(\ln x) = \sqrt{x}$ . Find  $f^{-1}(x)$ .

- (A)  $\ln x^2$
- (B)  $(\ln x)^2$
- (C)  $e^{\sqrt{x}}$
- (D)  $e^{x^2}$
- (E)  $\sqrt{\ln x}$

24. Simplify the sum  $\sum_{n=1}^{115} i^n$  if  $i = \sqrt{-1}$ .

- (A)  $-1$
- (B)  $0$
- (C)  $i$
- (D)  $1 - i$
- (E)  $-i$

25. An aquarium on a level table has a square base 10 in wide and is 8 in tall. When tilted, the water in it just covers one of the  $10\text{ in} \times 8\text{ in}$  ends but only three-fourths of the square bottom. What is the depth of the water when the bottom is again level?

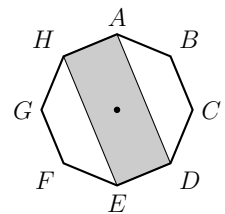
- (A) 2 in
- (B) 3 in
- (C) 3.25 in
- (D) 4 in
- (E) 6 in

26. Suppose the Earth is a perfect sphere and that there is a steel belt fitting snugly around it at the equator. If the length of the belt were increased by 10 feet, how far above the Earth would the belt be raised if it remained circular and centered around the Earth?

- (A) Less than 1 inch
- (B) Between 1 inch and 2 inches
- (C) Between 2 inches and 1 foot
- (D) Between 1 foot and 2 feet
- (E) More than 2 feet

27. In regular octagon  $ABCDEFGH$ , what is the fraction of the total octagon area found in rectangle  $ADEH$ ?

- (A)  $\frac{1}{2}$
- (B)  $\frac{7}{12}$
- (C)  $\frac{3}{5}$
- (D)  $\frac{2}{3}$
- (E)  $\frac{1}{3}$



28. Shannon takes her favorite number, adds 5 to it, multiplies the answer by 10, subtracts 20 from the result and then drops the final 0. If Shannon's (correct) answer is 9, what is her favorite number?
- (A) 5  
(B) 6  
(C) 7  
(D) 8  
(E) 9
29. If the standard order of operations is reversed (addition and subtraction are done first and exponentiation is done last), what is the value of  $2 \cdot 3^2 + 3$ ?
- (A) 21  
(B) 24  
(C) 39  
(D) 486  
(E) 7776
30. Simplify.  $\frac{\tan t - \sin t \cos t}{\tan t}$
- (A)  $\sin t$   
(B)  $\sin^2 t$   
(C)  $\cos t$   
(D)  $\cos^2 t$   
(E) 1
31. Different shades of pink, red, and white can be made by mixing whole numbers of quarts of red and white paint. Shades are different if the ratio of red to white is different. Find the number of different possible shades that can be made from at most 4 quarts of red and 5 quarts of white paint.
- (A) 15  
(B) 16  
(C) 17  
(D) 18  
(E) 19
32. How many strings of length 7 of  $a$ 's,  $b$ 's, and  $c$ 's are there such that all the  $a$ 's, if any, show up at the beginning? For example:  $aaaabcb$  is acceptable but  $abbbbab$  is not.
- (A) 254  
(B) 255  
(C) 256  
(D) 257  
(E) 258
33. Eric the Sheep is at the end of a line of 50 sheep waiting to be shorn. But being impatient, Eric sneaks up the line two places every time the shearer takes a sheep from the front to be shorn. So, for example, while the first sheep is being shorn, Eric moves ahead so that there are two sheep behind him in line. If at some point it is only possible for Eric to move one place, he does so. How many sheep get shorn **before** Eric?
- (A) 15  
(B) 16  
(C) 17  
(D) 18  
(E) 19
34. Find the values of  $c$  such that the trinomial  $x^2 + cx - 14$  can be factored over integers.
- (A)  $\{-15, -9, 9, 15\}$   
(B)  $\{-10, -4, 5, -19\}$   
(C)  $\{-14, -13, 13, 14\}$   
(D)  $\{-13, -5, 5, 13\}$   
(E)  $\{-15, -13, -9, -5, 5, 9, 13, 15\}$

35. From my balcony, I measure the angle of elevation to the top of the neighboring building to be  $30^\circ$ , and the angle of depression to the base to be  $45^\circ$ . If my (perpendicular) distance from the building is 20 m, how tall is the building in meters?

- (A)  $20 + \frac{20\sqrt{3}}{3}$
- (B)  $20 + \sqrt{3}$
- (C)  $20 + 20\sqrt{3}$
- (D)  $\frac{40}{3} + \frac{20\sqrt{3}}{3}$
- (E)  $\frac{40}{3} + \frac{40\sqrt{3}}{3}$

36. Simplify completely:

$$\sqrt[3]{\sqrt[5]{\sqrt{x^{40}}}}$$

- (A)  $\sqrt[30]{x^{40}}$
- (B)  $\sqrt[15]{x^{20}}$
- (C)  $x^{10} \sqrt[40]{x^{10}}$
- (D)  $x \sqrt[30]{x}$
- (E)  $x \sqrt[3]{x}$

37. If 10,  $2x$ , and  $3x$  are the first three terms of a geometric sequence, what is  $x$ ?

- (A)  $\frac{3}{2}$
- (B) 5
- (C)  $\frac{2}{3}$
- (D)  $\frac{15}{2}$
- (E)  $\frac{5}{3}$

38. Car A is 2 mi ahead of car B, which is going in the same direction. 8 min later car A is only 1 mi ahead of car B. On average, how much faster is car B traveling?

- (A) 5 mph
- (B) 7.5 mph
- (C) 10 mph
- (D) 15 mph
- (E) not enough info

39. A roll of paper towels contains 100 paper towels. The tube around which the towels are rolled is 2 cm in diameter. Including the tube, the whole roll is 14 cm in diameter. What is the diameter of the roll (and tube) when there only 50 paper towels left?

- (A) 6 cm
- (B) 7 cm
- (C) 8 cm
- (D) 10 cm
- (E) 12 cm

40. During shooting practice, a basketball player takes one step closer if she misses a shot, and one step farther away if she makes a shot. After a while, she notices she is two steps farther away than when she began. What is the most we can say about her shooting percentage  $P$  (i.e., shots made  $\div$  shots taken)?

- (A)  $25\% < P \leq 50\%$
- (B)  $P > 50\%$
- (C)  $P > 67\%$
- (D)  $P > 75\%$
- (E) not enough info