

Snow College Mathematics Contest

April 5, 2022

Senior Division: Grades 10-12

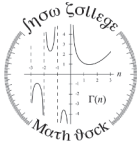
Form: T

Please read all instructions on this page very carefully.

1. Leave this booklet closed until you are instructed to begin.
2. Bubble your test version (Form T) on the upper left side of the bubble sheet. Make sure your bubble sheet has your name already printed (if not, return to the registration table).
3. This is a two hour examination consisting of 40 multiple choice problems. Bubble in the single best answer to each question you choose to answer clearly on the answer sheet. Completely erase any answers you wish to change.
4. Avoid random guessing as there is a penalty for wrong answers. There is no penalty for leaving a question blank. The formula for scoring the test is $\text{Score} = 40 + 4R - W$ where R and W denote the number right and wrong respectively. The possible scores range from 0 to 200.
5. Ties will be broken by the first discrepancy in the following five problems *in order*: 1, 2, 4, 9, 10. In the event of no discrepancies in those problems, the tie will be broken by a coin toss.
6. No calculators are allowed. Diagrams are not necessarily drawn to scale.
7. Do not talk or disrupt other test takers during the exam. Cell phones must be OFF (not just on silent or vibrate, but OFF). No ear buds are allowed.
8. Please raise your hand if you need scratch paper or a new pencil; a proctor will assist you.
9. The proctors have been advised to not answer questions pertaining to the exam.
10. If you have time we recommend you recheck your answers. If you finish early you may quietly turn your answer sheet in and leave. After the two hour time limit is up the proctors will call for all answer sheets; hand them in promptly.

After the test:

1. You may keep this test booklet and the pencils.
2. If you RSVP'd to spend time with one of our science departments for lunch, please meet them in the science center (GRSC) or library auditorium; otherwise lunch may be purchased at the Snow College Cafeteria or downtown. In any event, you should plan to be back at the LDS Institute by 1:00 p.m. for the scores and presentation of the awards.
3. Top individual and team scorers in each classification will receive prizes.
4. Thanks for coming; we hope you had fun and learned some math. Your instructors will be happy to work the problems for you. They will also be given copies of your answer sheets.



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Bubble in the single best choice for each question you choose to answer.

1. Evaluate the integral. $\int_0^{\infty} \frac{1}{1+x^2} dx$

- (A) $\frac{\pi}{2}$
- (B) $\frac{\pi}{4}$
- (C) $\frac{\pi}{3}$
- (D) $\sqrt{2}$
- (E) $\sqrt{3}$

2. Given three distinct two-digit numbers, what is the probability that two of them add up to the third one?

- (A) $\frac{1600}{90 \cdot 89 \cdot 88}$
- (B) $\frac{1600 \cdot 6}{90 \cdot 89 \cdot 88}$
- (C) $\frac{6400}{90 \cdot 89 \cdot 88}$
- (D) $\frac{6400 \cdot 6}{90 \cdot 89 \cdot 88}$
- (E) $\frac{1}{10}$

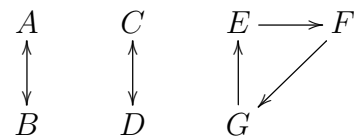
3. Solve for x (with $a > 0$): $2 \log_b x = 2 \log_b(1-a) + 2 \log_b(1+a) - \log_b\left(\frac{1}{a} - a\right)^2$

- (A) $x = a$
- (B) $x = \frac{1}{a}$
- (C) $x = a^2$
- (D) $x = 1 - a$
- (E) $x = \log_b a$

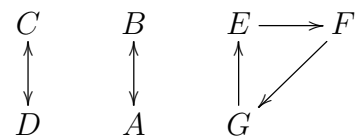
4. Towns A, B, and C are at the corners of a triangle with equal sides. A car travels at constant speed from A to B at 30 mph, from B to C at 40 mph, and from C back to A at 60 mph. What is the average speed for the round trip?

- (A) 40 mph
- (B) 43 mph
- (C) 45 mph
- (D) 48 mph
- (E) 50 mph

5. How many ways are there of permuting the letters in this arrow diagram such that the arrow relationships $A \leftrightarrow B$, $C \leftrightarrow D$, and $E \rightarrow F \rightarrow G \rightarrow E$ are preserved?



One acceptable permutation follows.



- (A) 6
- (B) 11
- (C) 24
- (D) 144
- (E) 504

6. Em and Jo are each at home and want to meet up at the SnoSpud which is 12 mi from Em's house and 15 mi from Jo's house (as the crow flies). If the SnoSpud makes a 60° angle with their houses, estimate how far apart they live (as the crow flies).
- (A) 8.9 mi
 (B) 13.7 mi
 (C) 17.6 mi
 (D) 19.2 mi
 (E) 27.0 mi
7. Zoe is holding a full container while walking. She begins to go up a ramp with slope m . What slope relative to the ramp must the container follow if she wants to keep the container's path parallel to the ground in front of the ramp?
- (A) $-\frac{1}{m}$
 (B) $-m$
 (C) $\frac{1}{m}$
 (D) m^2
 (E) $-\frac{1}{m^2}$
8. Find k so that the sum of the **squares** of the roots of $x^2 + 2x + k = 0$ is equal to 10.
- (A) -3
 (B) -2
 (C) -1
 (D) 0
 (E) 1
9. Find the area (in square units) of the triangle that is created by connecting the three points $(-2, 10)$, $(-1, -6)$, and $(8, 5)$.
- (A) 45.5
 (B) 50.5
 (C) 77.5
 (D) 80.5
 (E) 160.5
10. How many ways are there to rearrange the letters of "math is fun" such that the word "math" is contained in every rearrangement? Ignore spaces.
- (A) $9!$
 (B) $6!$
 (C) $\binom{9}{4}$
 (D) $6! \cdot 4!$
 (E) $\binom{9}{4} \cdot 4!$
11. If cars hold 5 passengers and charge \$29 a trip to the airport and vans hold 7 passengers and charge \$41, find the minimum cost to transport 49 people to the airport.
- (A) \$290
 (B) \$285
 (C) \$287
 (D) \$280
 (E) \$282
12. The matrix $A = \begin{bmatrix} a & 8 \\ -3 & b \end{bmatrix}$ is its own inverse, i.e., $AA^{-1} = A^{-1}A = AA = I$. Find $|a - b|$.
- (A) 4
 (B) 6
 (C) 8
 (D) 10
 (E) 12
13. Find the sum of the four fourth roots of 1.
- (A) 0
 (B) $\frac{1}{2}$
 (C) 1
 (D) 2
 (E) 4

14. A palindrome is a number that reads the same forward and backward, such as 2552. How many palindromes are there between 1000 and 9999 (including 9999)?
- (A) 90
 (B) 100
 (C) 121
 (D) 212
 (E) 3024
15. Computers use binary (base-two system). What is the value of the following binary number in our base-ten system? 1001011
- (A) 14
 (B) 75
 (C) 103
 (D) 125
 (E) 150
16. Many are familiar with the arithmetic mean where you add up all the values and divide by the number of values n . There is another mean called the *harmonic mean*:

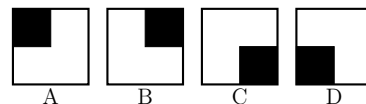
$$\frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}}$$

Find the harmonic mean of 10, 12, and 15.

- (A) $\frac{21}{1}$
 (B) 11
 (C) 12
 (D) $\frac{37}{3}$
 (E) $\frac{43}{3}$

17. Jaden has an unusual analog clock. The hour hand works like normal, but the minute hand travels backward (counter-clockwise) and completes one rotation in 30 minutes instead of 60 minutes (the minute hand points to the 12 on the hour and on the half-hour). How many times will the two hands be aligned in the four hours between 11:30 and 3:30?
- (A) 7
 (B) 8
 (C) 9
 (D) 10
 (E) 11

18. Plickers[®] is a quizzing method that uses a grid of pixels to convey information. A student shows the instructor a card which is read by a mobile device. Depending on the orientation of the pattern, one of four choices (A, B, C, D) is read. For example, a card using a 2×2 set of pixels could look like the following card in four orientations:



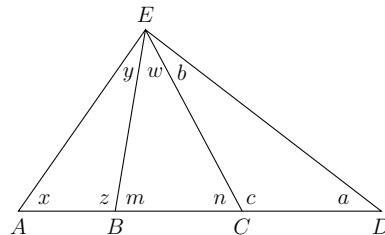
Each card must have a unique pattern for all four orientations. How many unique cards could be created using a 2×2 grid?

- (A) 1
 (B) 2
 (C) 3
 (D) 4
 (E) 5

19. Suppose that $a_n = \frac{1}{4}(a_{n-1}^2 + 3)$. Then if $a_0 = 0$, compute $\lim_{n \rightarrow \infty} a_n$.

- (A) ∞
 (B) 0
 (C) 1
 (D) 2
 (E) 4

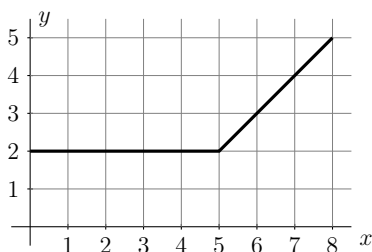
20. An additive partition of 6 is an ordered sum such as $3 + 2 + 1 = 6$ such that the addends appear in nonincreasing order. How many additive partitions of 6 are there? For example, the additive partitions of 3 are 3, $2 + 1$, and $1 + 1 + 1$.
- (A) 7
 (B) 8
 (C) 9
 (D) 10
 (E) 11
21. Snail racers (each 2 in long) are lined up with their tails touching the starting line and their bodies over (past) it. The race is 20 in from start to finish line and is over when any part of the snail crosses the finish line. The nose of one snail, Sluggard, moves 10 in in 2 min, then 5 in in the next 2 min, and so forth in an exponentially slowing way. Which value is *closest* to how long it will take Sluggard to finish the race?
- (A) Sluggard never finishes the race
 (B) 3 min
 (C) 4 min
 (D) 5 min
 (E) 6 min
22. A car averages 26.9 mpg. On a sunny day that the car is driven, the car averages 29 mpg. On a stormy day that the car is driven the car averages 22 mpg. What percent of the days is the car driven on a stormy day?
- (A) 30%
 (B) 40%
 (C) 50%
 (D) 60%
 (E) 70%
23. A yardstick is cut at 2 random places. What is the probability the sum of the two smaller pieces is longer than the largest piece?
- (A) $\frac{1}{4}$
 (B) $\frac{1}{3}$
 (C) $\frac{1}{2}$
 (D) $\frac{2}{3}$
 (E) $\frac{3}{4}$
24. Free-fall acceleration on planet RedOrbo is -4 m/s^2 . An explosion at ground level sends a rock flying vertically at an initial velocity of 12 m/s. How many seconds pass before the rock hits ground level? Ignore air resistance.
- (A) 3
 (B) 4
 (C) 6
 (D) 8
 (E) 12
25. In a general triangle ADE , lines EB and EC are drawn. Which of the following angle relationships is true?



- (A) $x + z = a + b$
 (B) $y + z = a + b$
 (C) $m + x = w + n$
 (D) $x + z + n = w + c + m$
 (E) $x + y + n = a + b + m$

26. The graph shown is of $f'(x)$. If $f(2) = 5$, use the graph to find $f(8)$.

- (A) 20.5
 (B) 21.5
 (C) 22.5
 (D) 23
 (E) 25



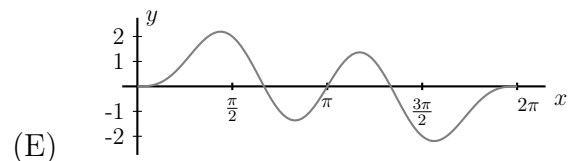
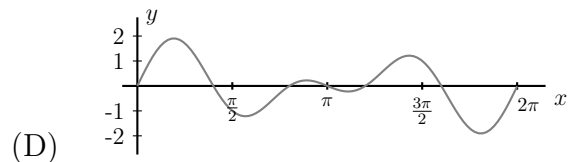
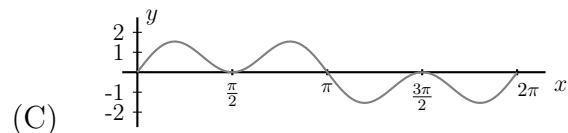
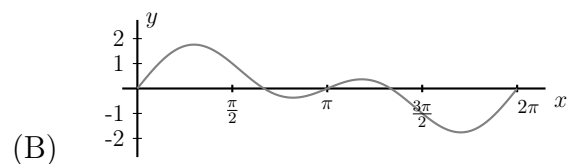
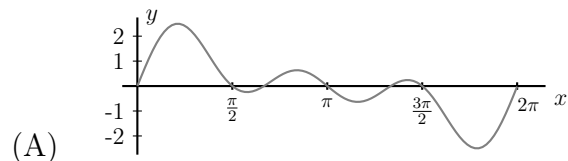
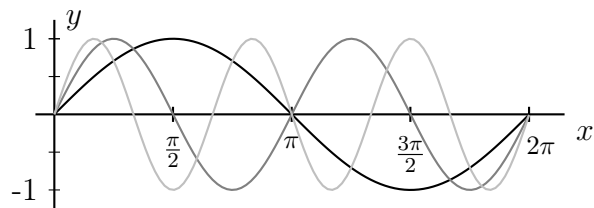
27. The points $(3, 3)$, $(6, 6)$, and $(10, -4)$ are on a circle. What is the area of the circle?

- (A) $\sqrt{25\pi}$
 (B) $\sqrt{29\pi}$
 (C) 25π
 (D) 27π
 (E) 29π

28. Let $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{2, 4, 6, 8\}$ both be subsets of $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Let X^C denote the complement in U of the subset X . What is $[[A \cap (A \cap B)^C] \cap B]^C$?

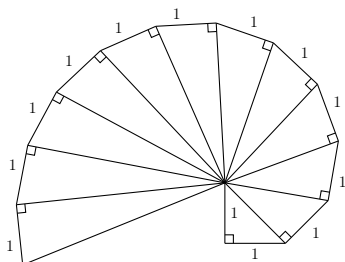
- (A) $\{\}$ or \emptyset
 (B) $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
 (C) $\{1, 3, 5, 7, 9\}$
 (D) $\{7, 8, 9\}$
 (E) $\{1, 3, 5\}$

29. Below you will find graphs of $y = \sin x$, $y = \sin 2x$, and $y = \sin 3x$ on the interval $[0, 2\pi]$. Which of the following graphs show $y = \sin x + \sin 2x + \sin 3x$?



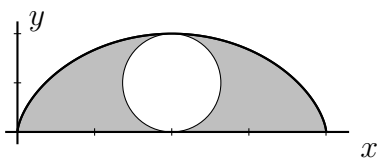
30. The *spiral of Theodorus* is built from an isosceles right triangle with legs of length 1. Additional right triangles are constructed with a leg of length one and the hypotenuse of the previous triangle serving as the other leg as illustrated below. Theodorus ended his construction with 16 triangles, although additional triangles may be added overlapping the prior triangles. If the first 100 triangles are constructed in this pattern, how many will have rational area?

- (A) 8
 (B) 9
 (C) 10
 (D) 12
 (E) 14



31. A cycloid is a curve traced out by a point on the rim of a wheel as it rolls without slipping along a flat surface. It is most easily defined by the set of parametric equations $x = r(t - \sin t)$, $y = r(1 - \cos t)$. It can be shown that the area under one arch of a cycloid is $3\pi r^2$ where r is the radius of the wheel. The shaded part of the area under the cycloid shown below is 24 in^2 . What is the area of the wheel?

- (A) 8 in^2
 (B) 9 in^2
 (C) 10 in^2
 (D) 12 in^2
 (E) 14 in^2

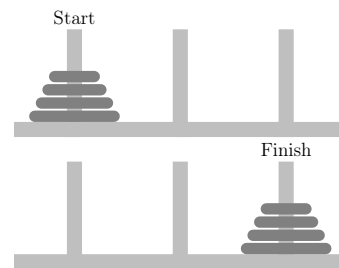


32. When each is given in Fahrenheit, the sum of the high and low temperatures for Ephraim on a particular day is 68. What is the sum of the high and low temperatures in Ephraim on the same day if each is given in Celsius? Hint: $F = \frac{9}{5}C + 32$.

- (A) $\frac{20}{9}$
 (B) $\frac{36}{5}$
 (C) 20
 (D) $\frac{772}{9}$
 (E) $\frac{772}{5}$

33. Four rings of different sizes are stacked on one of three posts in ascending order (smallest on top). You are able to move one ring at a time (taking the top ring from one post and moving it to another post), but you may never place a larger ring on a smaller ring. What is the minimum number of moves required to move the entire stack to a different post?

- (A) 12
 (B) 14
 (C) 15
 (D) 16
 (E) 17



34. Find the sum of all the real solutions to $|4 - |3 - |2 - |1 - x||| = 0$.

- (A) -2
 (B) -1
 (C) 0
 (D) 1
 (E) 2

35. For real numbers a and b , define an operation Δ as $a \Delta b = ab^2 - |a|$. Find $[(-2) \Delta 5] \Delta (-1)$.

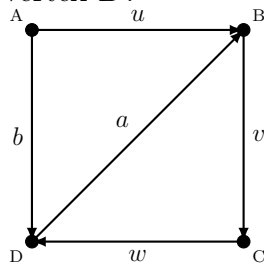
- (A) -104
 (B) -96
 (C) -53
 (D) 0
 (E) 96

36. A bucket of 200 marbles is 1% green and 99% red. How many red marbles need to be removed so that the percentage of red marbles in the bucket will be 98%?

- (A) 1
 (B) 2
 (C) 10
 (D) 50
 (E) 100

37. A *directed graph* is a configuration of directed edges (arrows) and vertices (dots) such that vertices are connected to other vertices by a directed edge. A *directed path* follows the direction of any number (repeats possible) of directed edges that connect two vertices. The length of the path is equal to the number of edges in the path. For example, a directed path of length 4 from vertex A to vertex B has the following edge sequence (u, v, w, a) . How many directed paths of length 48 are there from vertex A to vertex B ?

- (A) 0
 (B) 1
 (C) 2
 (D) 3
 (E) 4



38. Let A_1, A_2, \dots be a sequence of sets.

Define $\bigcup_{k=1}^{\infty} A_k = \{x \mid x \in A_k \text{ for some } k\}$ to be an infinite union $A_1 \cup A_2 \cup A_3 \cup \dots$, and $\bigcap_{k=1}^{\infty} A_k = \{x \mid \text{for every } k, x \in A_k\}$ to be an infinite intersection $A_1 \cap A_2 \cap A_3 \cap \dots$. If $A_k = \{k, k + 1, \dots\}$, then what is $\bigcap_{k=1}^{\infty} A_k$?

- (A) $\{1, 2, \dots\}$
 (B) $\{\infty\}$
 (C) $\{k + 1, k + 2, \dots\}$
 (D) $\bigcup_{k=1}^{\infty} A_k$
 (E) \emptyset

39. Tim is playing a game of dice. He must pay \$2 to roll a pair of fair six-sided dice. If he rolls doubles, he wins \$9. What are the expected net winnings per game?

- (A) $-\$0.50$
 (B) $-\$0.33$
 (C) $\$0.50$
 (D) $\$1.17$
 (E) $\$3.00$

40. Find the limit.

$$\lim_{x \rightarrow \infty} \frac{16x^2 - x^3}{2^x}$$

- (A) $-\infty$
 (B) -1
 (C) 0
 (D) 1
 (E) ∞