

Snow College Mathematics Contest

April 1, 2014

Senior Division: Grades 10-12

Form: **T**

Bubble in the single best choice for each question you choose to answer.

- Simplify. $4^{\log_2(2^{1/4} \cdot 2^{1/8} \cdot 2^{1/16} \dots)}$
 - 1
 - $\sqrt{2}$
 - 2
 - $2\sqrt{2}$
 - 4
- The Pauli spin matrices σ_1 , σ_2 , and σ_3 appear in quantum mechanics. They are
$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$
The inverse of a matrix A is A^{-1} such that $AA^{-1} = A^{-1}A = I$. What is $(\sigma_2)^{-1}$?
 - σ_1
 - σ_2
 - σ_3
 - $-\sigma_2$
 - I
- A sprinkler in the corner of a large square lawn sprays a radius of 16 ft. Approximately how many square feet of lawn are watered by the sprinkler?
 - 800 ft²
 - 400 ft²
 - 256 ft²
 - 200 ft²
 - 64 ft²
- What is the output of the following BASIC computer program?

```
10 dim V(7) : rem dimension array
20 R = 1
30 V(0) = 1 : print V(0)
40 V(1) = 2 * R : print V(1)
50 for n = 2 to 7
60 V(n) = (2 * pi * R^2/n) * V(n-2)
70 print V(n)
80 next n
```

 - velocity in a collision of n objects
 - viscosity of a liquid at temperature n
 - n th vibrational mode of an oscillator
 - n th vaporization state of a gas
 - volume of an n -dim unit sphere
- One solution of $4x^2 + bx - 3 = 0$ is $\frac{9}{4}$. Find b and the other solution.
 - $b = -\frac{9}{4}$; $x = \frac{1}{3}$
 - $b = -9$; $x = -\frac{4}{3}$
 - $b = -\frac{23}{3}$; $x = -\frac{1}{3}$
 - $b = \frac{23}{3}$; $x = -\frac{1}{3}$
 - $b = \frac{9}{4}$; $x = \frac{1}{3}$
- A man born in the first half of the nineteenth century was x years old in the year x^2 . In what year was he born?
 - 1806
 - 1812
 - 1825
 - 1836
 - 1849

7. Gaussian integers are complex $a + bi$ with $a, b \in \mathbb{Z}$. Some ordinary primes, such as 3, 7, 11, 19, 23, 31, 43, ... are also prime in the Gaussian integers; but 5 is not a Gaussian prime because $5 = (2+i)(2-i)$. Which ordinary primes a are also Gaussian primes?

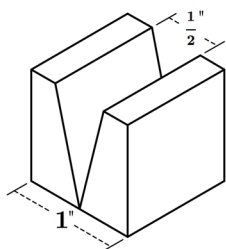
- (A) $a = 2^n + 1$
- (B) $a = 2n - 1$
- (C) $a = b$
- (D) $a = 3n + 1$
- (E) $a \equiv 3 \pmod{4}$

8. Find the sum of the three cube roots of 1.

- (A) 0
- (B) $\frac{1}{2}$
- (C) 1
- (D) 2
- (E) 3

9. A wedge is removed from the center of a cube as shown. How much of the original volume of the cube remains?

- (A) $\frac{3}{5}$
- (B) $\frac{1}{2}$
- (C) $\frac{3}{4}$
- (D) $\frac{\sqrt{2}}{2}$
- (E) $\frac{4}{5}$



10. How many factors does 1 000 000 have?

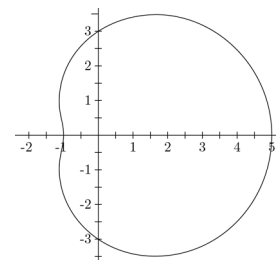
- (A) 48
- (B) 49
- (C) 50
- (D) 51
- (E) 52

11. Which of the following fractions cannot be written as a terminating decimal?

- (A) $\frac{7}{8}$
- (B) $\frac{11}{250}$
- (C) $\frac{11}{125}$
- (D) $\frac{37}{768}$
- (E) $\frac{99}{256}$

12. Which polar equation best represents the graph for $0 \leq \theta \leq 2\pi$?

- (A) $r = 3 + 2 \cos \theta$
- (B) $r = 2 + 3 \cos \theta$
- (C) $r = 2 + 3 \sin \theta$
- (D) $r = 3 + 2 \sin \theta$
- (E) $r = 3 + 3 \sin \theta$



13. What is the length of the curve?

$$y = \frac{2}{3}(x - 1)^{\frac{3}{2}}, \quad 1 \leq x \leq 4$$

- (A) $\frac{12\sqrt{3}}{5}$
- (B) 9
- (C) $\frac{14}{3}$
- (D) 12
- (E) $2\sqrt{3}$

14. If the graphs of $2y + x + 3 = 0$ and $3y + ax + 2 = 0$ are to meet at right angles, then what is a ?

- (A) -6
- (B) $-\frac{2}{3}$
- (C) $-\frac{1}{2}$
- (D) $\frac{3}{2}$
- (E) 6

15. The empty set is a subset of every set. A set is also a subset of itself. The *power set* of a set S , $\mathcal{P}(S)$, is the set of all subsets of S . How many elements are there in $\mathcal{P}(S)$ if $S = \{R, O, Y, G, B, I, V\}$?
- (A) 124
 (B) 125
 (C) 126
 (D) 127
 (E) 128
16. The perimeter of a certain right triangle is $12 + 8\sqrt{3}$. The sum of the squares of all three of its sides is 294. Find its area.
- (A) $11 + \sqrt{3}$
 (B) $\frac{147}{4}$
 (C) $10\sqrt{3}$
 (D) $6\sqrt{3}$
 (E) $24\sqrt{3}$
17. This problem involves numbers written in two bases we'll call base A and base B . What is the base 10 value of $A + B$ if $31_A = 2_A \cdot 17_A$ and $44_B = 3_B \cdot 13_B$?
- (A) 20
 (B) 18
 (C) 14
 (D) 11
 (E) 8
18. $x^{x^{x^{\dots}}} = 2$ is true for what value of x ?
- (A) 2
 (B) $\sqrt{2}$
 (C) $\sqrt[4]{2}$
 (D) $2\sqrt{2}$
 (E) ∞
19. An inlet pipe can fill an empty tank by itself in 2 hours and an outlet pipe can drain the same full tank by itself in 5 hours. If the tank is half full when both valves are opened how long will it take to fill the tank?
- (A) 1 h, 50 min
 (B) 1 h, 40 min
 (C) 1 h, 28 min
 (D) 1 h, 18 min
 (E) 1 h, 6 min
20. Where does the line tangent to the sine function at $(\pi, 0)$ intersect the y -axis?
- (A) $(0, 0)$
 (B) $(0, \pi)$
 (C) $(0, 1)$
 (D) $(0, \frac{\pi}{2})$
 (E) $(0, 2\pi)$
21. At her birthday party Mrs. B was asked her age. She replied that the total of her age and the age of her husband is 140. Then she added "My husband is twice the age I was when he was my age." What is the product of Mr. and Mrs. B's ages?
- (A) 4500
 (B) 4756
 (C) 4800
 (D) 4875
 (E) 4891
22. A triangle has sides of lengths 8.1 and 1.4. What is the length of the third side, if it is an even integer?
- (A) 2
 (B) 4
 (C) 6
 (D) 8
 (E) 10

23. Find $b - a$ if (a, b) is the solution to the system of equations.

$$\begin{cases} \pi a + (\pi + e)b = \pi + 2e \\ (\pi + 3e)a + (\pi + 4e)b = \pi + 5e \end{cases}$$

- (A) -3
 (B) -1
 (C) 0
 (D) 1
 (E) 3

24. Simplify the expression: $\frac{\tan t - \sin t \cos t}{\tan t}$

- (A) $\sin t$
 (B) $\cos t$
 (C) $\sin^2 t$
 (D) $\cos^2 t$
 (E) 1

25. Randomly choose k digits (with repetitions allowed) from $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. What is the probability that 0 will *not* be chosen?

- (A) $\frac{1}{k}$
 (B) $\frac{1}{10}$
 (C) $\frac{k}{k-1}$
 (D) $\left(\frac{1}{10}\right)^k$
 (E) $\left(\frac{9}{10}\right)^k$

26. How many minutes after noon is the first time that the hour and minute hands are pointing in exactly opposite directions?

- (A) 32
 (B) $32\frac{1}{2}$
 (C) $32\frac{7}{12}$
 (D) $32\frac{8}{11}$
 (E) 33

27. In one lottery there is a bubble sheet of numbers from 01–60 and a ticket is completed by filling in any five of those numbers. If you randomly pick the five numbers, what are the chances you will win the jackpot?

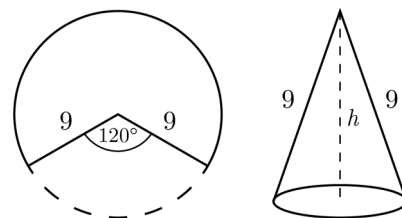
- (A) $\frac{5!}{60!}$
 (B) $\frac{5!}{55!}$
 (C) $\frac{5! \cdot 55!}{60!}$
 (D) $\frac{1}{5!}$
 (E) $\frac{50!}{55!}$

28. Let the bottom edge of a rectangular mirror on a vertical wall be parallel to and h feet above the level floor. If a person with eyes t feet above the floor is standing erect at a distance d feet from the mirror, what is the relationship among h , d , and t if the person can just see his own feet in the mirror?

- (A) $t = 2h$ and d doesn't matter
 (B) $t = 4d$ and h doesn't matter
 (C) $h^2 + d^2 = \frac{t^2}{4}$
 (D) $t - h = d$
 (E) $(t - h)^2 = 4d$

29. A piece of paper has the shape of the larger circular sector with dimensions and angles as shown. This paper is suitably folded to form the vertical cone shown at right. Find the height h of the cone.

- (A) 3
 (B) $3\sqrt{5}$
 (C) $3\sqrt{2}$
 (D) $9\sqrt{2}$
 (E) 9



30. It's Sophie's birthday! Sophie Germain, famous, self-taught woman mathematician was born April 1, 1776. The identity named after her says which of the following is equivalent to $x^4 + 4y^4$?

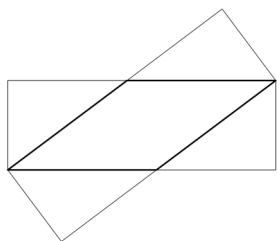
- (A) $(x + y)(x + y)(x - y)(x - y)$
- (B) $(x^2 + 2xy + 2y^2)(x^2 - 2xy + 2y^2)$
- (C) $(x + y)^2(x + 2y)^2$
- (D) $(x + y)^2(x - y)(x + 4y)$
- (E) $(x - y)^2(x + y)(x + 4y)$

31. What is the real number equivalent of i^{2i} ?

- (A) e^{-1}
- (B) $e^{-\frac{\pi}{2}}$
- (C) $e^{-\pi}$
- (D) $e^{-2\pi}$
- (E) No real number equivalent exists

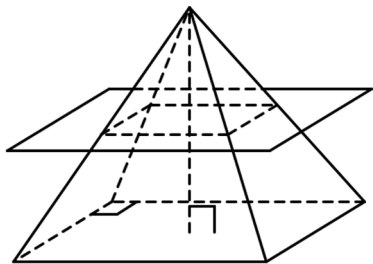
32. A copy of a $6\text{ cm} \times 18\text{ cm}$ rectangle is placed on top of the original rectangle as shown. What is the area of the parallelogram formed by the intersection?

- (A) 54 cm^2
- (B) $12\sqrt{34}\text{ cm}^2$
- (C) $18\sqrt{10}\text{ cm}^2$
- (D) $36\sqrt{2}\text{ cm}^2$
- (E) 60 cm^2



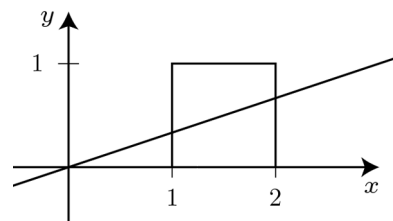
33. A right square pyramid is cut by a plane parallel to its base halfway up the altitude, creating a smaller version of the pyramid. What is the ratio of the volume of the small pyramid to that of the original?

- (A) 1 : 2
- (B) 1 : 4
- (C) 1 : 6
- (D) 1 : 8
- (E) 1 : 10



34. Find the equation of the line through the origin that bisects the area of the unit square shown.

- (A) $y = \frac{1}{3}x$
- (B) $y = \frac{1}{2}x$
- (C) $y = \frac{2}{3}x$
- (D) $y = \frac{3}{2}x$
- (E) $y = 2x$



35. Let f be a function satisfying

$$f(xy) = \frac{f(x)}{y}$$

If $f(500) = 3$, what is $f(600)$?

- (A) $\frac{2}{5}$
- (B) $\frac{5}{6}$
- (C) $\frac{6}{5}$
- (D) $\frac{5}{2}$
- (E) $\frac{18}{5}$

36. Positive integers a , b , c , with no common factor greater than 1, exist such that $a \log_{200} 5 + b \log_{200} 2 = c$. What is the sum $a + b + c$?

- (A) 6
- (B) 7
- (C) 8
- (D) 9
- (E) 10

37. If $\sin x = 3 \cos x$, what is $\sin x \cos x$?

- (A) $\frac{1}{6}$
- (B) $\frac{1}{5}$
- (C) $\frac{2}{9}$
- (D) $\frac{1}{4}$
- (E) $\frac{3}{10}$

38. Five girls wish to give each other gifts. Each girl puts her name in a hat and then each draws a name from the hat. In how many ways can the names be drawn from the hat so that no one draws her own name?

- (A) 24
- (B) 25
- (C) 44
- (D) 86
- (E) 120

39. Which is true if the lines with equations $y = 2x + b$ and $y = mx - 6$ intersect at a point on the x -axis?

- (A) $mb = 12$
- (B) $mb + 12 = 0$
- (C) $m = 3b$
- (D) $m + 3b = 0$
- (E) $3m = b$

40. What is the area shared by two intersecting circles of radius 1 passing through each other's center?

- (A) $\frac{\sqrt{3}}{2}$
- (B) $\frac{2\pi}{3} - \frac{\sqrt{3}}{2}$
- (C) $\frac{\sqrt{3}}{2} + \frac{\pi}{18}$
- (D) $\frac{\pi}{4}$
- (E) $\frac{\pi}{3}$

